Simultaneous Radiative and Convective Heat Transfer in an Absorbing, Emitting, and Scattering Medium in Slug Flow Between Parallel Plates

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The problem of coupled radiation and convection to a medium in slug flow between infinite, parallel plates was formulated in terms of discrete fluxes. Steady state, uniform heat input, and gray boundaries were assumed. The effects of absorption, emission, scattering, and flow were taken into account. For the general case where all these effects are present, it was necessary to approximate the fourth-order temperature term by a truncated Taylor series. This approximation was not necessary for the special case where conductivity of the medium is negligible.

Approximate, analytical solutions in closed form for profiles of temperature, radiant fluxes, and heat fluxes were obtained for both the special case and the general case. Sample results are presented, showing the effects of convection, absolute temperature, absorption coefficient, scattering coefficient, and wall emissivities. The two cases of equal wall temperatures and unequal wall temperatures were both examined.

It was found that for a medium with properties approximately representative of those for air carrying a suspension of dust or mist, radiation can account for 60 to 95% of total heat transfer in the temperature range of 900° to 2,800°R. It was also found that for some situations the scattering phenomenon can have a much greater effect on heat transfer than the absorption and emission phenomena.

Radiative heat transfer is inherently different from conductive and convective heat transfer. While the latter two processes transfer the kinetic energy of molecules by direct molecular collision or transport, the radiative process transfers energy in the form of electromagnetic waves and does not require direct contact of molecules in the transfer medium. Radiation becomes coupled with conduction and convection only when the medium is able to convert electromagnetic energy into molecular-kinetic energy or vice versa, that is if the medium is absorbing and/or emitting.

Recently there has been increased interest in such cases of coupled, simultaneous heat transfer. The phenomenon often occurs in gases, water vapor, gases carrying liquid or solid particles, and plasmas. Applications are found in gas-cooled nuclear reactors, chemical rockets, fusion experiments, magnetohydrodynamic generators, fluidized beds, and ablative cooling of space vehicles.

An excellent review of this field of heat transfer is presented by Cess (*1*). Among the more notable recent works is that of Viskanta and Grosh (*2*), who analyzed the case of radiative and conductive heat transfer in one-dimensional medium with absorption and emission but without scattering and without flow. A closed-form solution was not found, but results were obtained by converting the appropriate integro-differential equation into a nonlinear integral equation which was then solved by iteration. Goulard and Goulard (*3*) treated the same problem and also obtained results by iteration. Viskanta and Grosh (*4*) in another paper considered the case of boundary-layer flow past a wedge with radiation and conduction. They utilized the Rosseland approximation for optically thick media and obtained results by numerical integration. However as the authors pointed out, "the approximation fails in the vicinity of the surface since it does not properly take into account radiation leaving from the surface."

The authors partially corrected for this error by using a different value for their numerical constant in the vicinity of the wall. Recently Einstein (5) studied the case of an absorbing medium flowing between parallel plates, computing results numerically by the finite difference method. Viskanta (6) has also studied the same case. By using a three-term Taylor series to approximate the temperature function the author reduced the integro-differential equation to a nonlinear differential equation which can be solved numerically by iteration.

In all the references mentioned above the medium was assumed to be nonscattering. However it has been found (7, 8) that in many situations the gross scattering phenomenon can have a dominant effect on radiative transfer. The purpose of this study was to analyze the heat transfer problem for media which scatter as well as absorb and emit thermal radiation, in a situation where conduction and flow effects are present. For simplicity in an already complex problem the following limitations were imposed: gray medium with constant physical properties, steady state with uniform heat input, parallel plates geometry with gray boundaries, slug flow, and isotropic scattering. An approximate, analytical solution was sought rather than exact numerical results in the anticipation that the usefulness of a solution is closed form would offset the disadvantage of slight inexactness.

The discrete-flux method of Hamaker (9) was used in this analysis. This method has been used successfully to treat the case of radiation and conduction without flow by Larkin and Churchill (7) and by Chen and Churchill (8). In the present study this analysis is extended to the case where slug flow occurs.

DERIVATION OF EQUATIONS

The system of interest is represented in Figure 1. If the monochromatic intensity of radiant energy traveling in a direction $\vec{\Omega}$ is represented by i_λ, then the total flux of radiant energy passing forward through a unit plane which is perpendicular to the normal vector $\vec{n}$ is given by the integral

$$i_+ = \int_{\lambda=0}^{\infty} 2\pi \int_{\theta=0}^{\pi/2} i_\lambda \cos\theta \sin\theta \, d\theta \, d\lambda \qquad (1)$$

Similarly the total flux passing backward (in negative $\vec{n}$ direction) through the unit area is

$$i_- = \int_{\lambda=0}^{\infty} 2\pi \int_{\theta=\pi}^{3\pi/2} i_\lambda \cos\theta \sin\theta \, d\theta \, d\lambda \qquad (2)$$

and the net flux passing through the area is

$$i_n = i_+ - i_- \qquad (3)$$

which has units of energy per unit area per unit time. Thus the net flux of radiation in the y direction i_y is equal to i_n for $\vec{n}$ taken in the direction of the positive y axis. Similarly the net flux in the x direction i_x is equal to i_n for $\vec{n}$ taken in the direction of the positive x axis. In these terms the steady state, two-dimensional equation for conservation of energy with conductive and radiative heat transfer can be written as follows:

$$k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] - \rho\, C_p\left[\frac{v_x\,\partial T}{\partial x} + \frac{v_y \partial T}{\partial y}\right] - \left[\frac{\partial i_x}{\partial x} + \frac{\partial i_y}{\partial y}\right] = 0 \qquad (4)$$

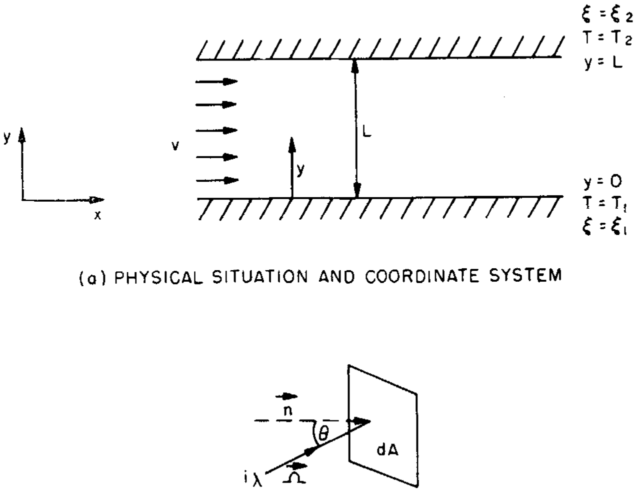
For slug flow

$$v_y = 0,\ v_x = v \text{ (a constant)}$$

$$\rho\, C_p\left[v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y}\right] = \rho\, C_p\, v\frac{\partial T}{\partial x}$$

Under fully developed conditions the temperature profile maintains the same shape along the channel, and with uniform heat input the fluid gains sensible heat at a constant rate as it progresses along the channel. These two conditions necessarily require that the axial temperature gradient $\partial T/\partial x$ be constant; thus

$$\rho\, C_p\, v\frac{\partial T}{\partial x} = C, \text{ a constant}$$



(a) PHYSICAL SITUATION AND COORDINATE SYSTEM

(b) COORDINATE SYSTEM FOR RADIANT FLUX

Fig. 1. Coordinate systems.

If it is further assumed that heat transfer in the longitudinal direction is negligible compared with lateral heat transfer, then

$$\frac{\partial^2 T}{\partial x^2} << \frac{\partial^2 T}{\partial y^2}$$

$$\frac{\partial i_x}{\partial x} << \frac{\partial i_y}{\partial y}$$

and Equation (4) reduces to

$$k\frac{\partial^2 T}{\partial y^2} - \frac{\partial i_y}{\partial y} = \rho C_p v\frac{\partial T}{\partial x} \qquad (5)$$

The net flux i_y is the difference between a forward flux and a backward flux, defined by Equations (1) and (2) respectively, with $\vec{n}$ taken in the direction of the y axis. Let the forward flux be denoted by I and the backward flux by J. As the flux I travels an incremental length ∂y, it loses energy by absorption and scattering. It gains energy by having part of the backward flux scattered into the forward direction, and it gains energy by thermal emission from the incremental element. For an isotropic, gray medium this transport process can be described by the following equation:

$$\frac{\partial I}{\partial y} = -(a+s)I + (s)J + a\sigma T^4 \qquad (6)$$

The analogous equation for J is

$$\frac{\partial J}{\partial y} = + (a+s)J - (s)I - a\sigma T^4 \qquad (7)$$

Then

$$\frac{\partial i_y}{\partial y} = \frac{\partial}{\partial y}(I - J) = \frac{\partial I}{\partial y} - \frac{\partial J}{\partial y} = -a(I+J) + 2a\sigma T^4 \qquad (8)$$

Substitution into Equation (5) gives

$$k\frac{\partial^2 T}{\partial y^2} + a(I+J) - 2a\sigma T^4 = \rho C_p v \frac{\partial T}{\partial x} \qquad (9)$$

Equations (6), (7), and (9) comprise a system of nonlinear, fourth-order differential equations which describes the physical phenomenon. The necessary boundary conditions are

$$T \text{ (at } y = 0) = T_1 \qquad (10)$$

$$T \text{ (at } y = L) = T_2 \qquad (11)$$

$$I_1 \text{ (at } y = 0) = \epsilon_1 \sigma T_1^4 + (1 - \epsilon_1) J_1 \qquad (12)$$

$$J_2 \text{ (at } y = L) = \epsilon_2 \sigma T_2^4 + (1 - \epsilon_2) I_2 \qquad (13)$$

The heat fluxes at any point would then be

$$q_c = -k\frac{\partial T}{\partial y} \qquad (14)$$

$$q_r = I - J \qquad (15)$$

$$q = q_c + q_r \qquad (16)$$

It is of interest to note that the occurrence of uniform heat generation or heat sinks within the medium can be treated by this same formulation. The only change necessary is to redefine the constant C as

$$C = \rho v C_p \frac{\partial T}{\partial x} - G$$

SPECIAL CASE

Before the equations for the general case are solved, it is appropriate to examine the special limiting case where

$$k = 0$$
$$v = 0$$
$$s = 0$$
$$\epsilon_1 = \epsilon_2 = 1 \qquad (17)$$

Exact numerical results are available (*10*) for this special case, and they provide a means of checking the present formulation.

Under the conditions (17) Equations (6), (7), and (9) reduce to

$$\frac{\partial I}{\partial y} = a(-I + \sigma T^4) \qquad (18)$$

$$\frac{\partial J}{\partial y} = a(J - \sigma T^4) \qquad (19)$$

$$(I + J) - 2\sigma T^4 = 0 \qquad (20)$$

The system of equations is now linear in T^4 and can be solved to obtain

$$T^4 = C_1 y + C_2 \qquad (21)$$

$$I = \frac{\sigma}{a} C_1 (ay - 1) + \sigma C_2 + C_3 e^{-ay} \qquad (22)$$

$$J = \frac{\sigma}{a} C_1 (ay + 1) + \sigma C_2 + C_4 e^{ay} \qquad (23)$$

where C_1, C_2, C_3, C_4 are constants. Under the stipulation of zero conductivity the temperatures need no longer be continuous at the boundaries, so that boundary conditions (10) and (11) are not applicable. The governing condition is now

$$q = q_r = \text{constant everywhere} \qquad (24)$$

Substituting Equations (21), (22), and (23) into Equation (20) one obtains

$$C_3 e^{-ay} + C_4 e^{ay} = 0 \qquad (25)$$

For this to be true at all values of y

$$C_3 = C_4 = 0 \qquad (26)$$

and it follows that

$$q_r = I - J = -2\frac{\sigma}{a} C_1 \qquad (27)$$

At the boundaries

$$q_r (y = o) = \sigma T_1^4 - J_1 = \sigma T_1^4 - \frac{\sigma}{a} C_1 - \sigma C_2 \qquad (28)$$

$$q_r (y = L) = I_2 - \sigma T_2^4 = \frac{\sigma}{a} C_1 (aL - 1) + \sigma C_2 - \sigma T_2^4 \qquad (29)$$

Equating Equation (27) with Equations (28) and (29) one obtains

$$C_1 = -a\left[T_1^4 - T_1^4\left(\frac{aL+1}{aL+2}\right) - T_2^4\left(\frac{1}{aL+2}\right)\right] \qquad (30)$$

$$C_2 = T_1^4\left(\frac{aL+1}{aL+2}\right) + T_2^4\left(\frac{1}{aL+2}\right) \qquad (31)$$

and so

$$q_r = \frac{2\sigma}{aL+2}(T_1^4 - T_2^4) \qquad (32)$$

SOLUTION FOR GENERAL CASE

To obtain a closed-form solution to the system of Equations (6), (7), and (9) it is necessary to represent the fourth-order temperature term by some approximate linear function. One method is to expand T^4 in a Taylor series about some arbitrary temperature T_o and then neglect all terms beyond the second:

$$T^4 = T_o^4 + (T - T_o)\left[\frac{\partial T^4}{\partial T}\right]_{T_o} + \cdots$$
$$= (4T_o^3)T - 3T_o^4 \qquad (33)$$

Substituting into Equations (6), (7), and (9) and clearing one obtains the following system of three uncoupled, linear, differential equations:

$$\frac{\partial^4 T}{\partial y^4} - \delta^2 \frac{\partial^2 T}{\partial y^2} = -\frac{\gamma^2}{k} v\rho C_p \frac{\partial T}{\partial x} \qquad (34)$$

$$\frac{\partial^4 I}{\partial y^4} - \delta^2 \frac{\partial^2 I}{\partial y^2} = -4\sigma T_o^3 \frac{\gamma^2}{k} v\rho C_p \frac{\partial T}{\partial x} \qquad (35)$$

$$\frac{\partial^4 J}{\partial y^4} - \delta^2 \frac{\partial^2 J}{\partial y^2} = -4\sigma T_o^3 \frac{\gamma^2}{k} v\rho C_p \frac{\partial T}{\partial x} \qquad (36)$$

where

$$\gamma^2 = a(a + 2s) \qquad (37)$$

$$\delta = \left[\frac{8a\sigma T_o^3}{k} + a(a + 2s) \right]^{1/2} \qquad (38)$$

The solutions for the temperature and radiant flux profiles are then obtained as

$$T = A_1 + A_2 y + \frac{C}{2k(1+\alpha)} \cdot y^2 + A_3 e^{-\delta(L-y)} + A_4 e^{-\delta y} \qquad (39)$$

$$I = 4\sigma T_o^3 A_1 - (1/2)\,\alpha k A_2 + \frac{C\alpha}{2a(1+\alpha)} - 3\sigma T_o^4 + \left[4\sigma T_o^3 A_2 - \frac{C\alpha}{2(1+\alpha)} \right] y + \frac{2\sigma T_o^3 C}{k(1+\alpha)} \cdot y^2 + \frac{4\sigma T_o^3(\beta - 1)}{\alpha} A_3 e^{-\delta(L-y)} - \frac{4\sigma T_o^3(\beta+1)}{\alpha} A_4 \cdot e^{\delta y} \qquad (40)$$

$$J = 4\sigma T_o^3 A_1 + (1/2)\alpha k A_2 + \frac{C\alpha}{2a(1+\alpha)} - 3\sigma T_o^4 + \left[4\sigma T_o^3 A_2 + \frac{C\alpha}{2(1+\alpha)} \right] y + \frac{2\sigma T_o^3 C}{k(1+\alpha)} \cdot y^2 - \frac{4\sigma T_o^3(\beta+1)}{\alpha} A_3 \cdot e^{-\delta(L-y)} + \frac{4\sigma T_o^3(\beta-1)}{\alpha} A_4 \cdot e^{-\delta y} \qquad (41)$$

where

$$\alpha = \frac{8\sigma T_o^3}{k\,(a + 2s)} \qquad (42)$$

$$\beta = \frac{\delta}{a + 2s} \qquad (43)$$

The convective heat flux at any point y due to molecular conduction is

$$q_c = -k\frac{\partial T}{\partial y} = -kA_2 - \frac{C}{1+\alpha} y - kA_3\delta e^{-\delta(L-y)} + kA_4\delta e^{-\delta y} \qquad (44)$$

and the radiant and total heat fluxes are given by Equations (15) and (16) respectively.

In the above solutions A_1, A_2, A_3, and A_4 are constants which are determined by boundary conditions (10) through (13). The resultant expressions for these constants are so lengthy that they will not be given here. Instead simplified approximate expressions can be obtained, as follows.

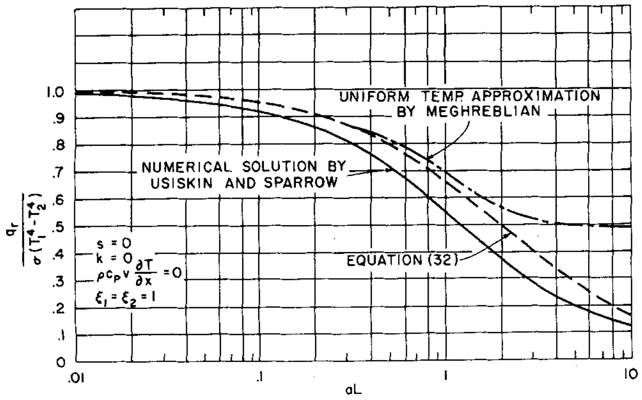


Fig. 2. Radiative transfer for limiting case of no flow, conduction, or scattering.

Noting that for most cases of interest

$$\delta = 0 \ (10 \text{ to } 100)$$

and

$$e^{\delta L} \cong 0 \ (10^{50})$$

one can then assume that

$$F \pm e^{\delta L} = \pm e^{\delta L}$$
$$e^{-\delta L} = 0$$

where F is any expression with value less than $0(10^{40})$ With this simplification the boundary constants are determined to be

$$A_1 = \{f_3 f_8(f_9 - T_2) + (f_1 - T_1 f_4)(Lf_8 - f_7) + f_5 f_3\} \div \{Lf_8(f_2 - f_4) + f_3(f_6 - f_8) + f_7(f_4 - f_2)\} \qquad (45)$$

$$A_2 = \{f_8(T_1 - T_2 + f_9)(f_4 - f_2) + (f_1 - T_1 f_2)(f_6 - f_8) + (f_5 - T_1 f_6)(f_4 - f_2)\} \div \{Lf_8(f_2 - f_4) + f_3(f_6 - f_8) + f_7(f_4 - f_2)\} \qquad (46)$$

$$A_3 = \{L[f_6(T_1 f_2 - f_1) - (f_4 - f_2)(f_5 - T_1 f_6)] - f_3[f_6(T_1 - T_2 + f_9) + f_5 - T_1 f_6] + f_7[(f_4 - f_2)(T_2 - T_1 - f_9) + f_1 - T_1 f_2]\} \div \{Lf_8(f_2 - f_4) + f_3(f_6 - f_8) + f_7(f_4 - f_2)\} \qquad (47)$$

$$A_4 = \{Lf_8(T_1 f_2 - f_1) - f_3[(f_5 - T_1 f_6) + f_8(T_1 - T_2 + f_9)] + f_7(f_1 - T_1 f_2)\} \div \{Lf_8(f_2 - f_4) + f_3(f_6 - f^8) + f_7(f_4 - f_2)\} \qquad (48)$$

where

$$f_1 = \epsilon_1 \left[\sigma(T_1^4 + 3T_o^4) - \frac{C\alpha}{2a(1+\alpha)} \right]$$

$$f_2 = 4\epsilon_1 \sigma T_o^3$$

$$f_3 = 4\sigma T_o^3 \frac{(\epsilon_1 - 2)}{(a + 2s)}$$

$$f_4 = \frac{4\sigma T_o^3}{\alpha}[\beta(\epsilon_1 - 2) - \epsilon_1]$$

$$f_5 = \epsilon_2 \sigma(T_2^4 + 3T_o^4) - \frac{C\epsilon_2}{2(1+\alpha)} \left(\frac{\alpha}{a} + \frac{4\sigma T_o^3 L^2}{k} \right) + \frac{C\alpha L(\epsilon_2 - 2)}{2(1+\alpha)} \qquad (49)$$

$$f_6 = 4\epsilon_2 \sigma T_o^3$$

$$f_7 = 4\sigma T_o^3 \left[\epsilon_2 L - \frac{\epsilon_2 - 2}{a + 2s} \right]$$

$$f_8 = \frac{4\sigma T_o^3 \beta(\epsilon_2 - 2)}{\alpha} - \epsilon_2$$

$$f_9 = \frac{CL^2}{2k(1+\alpha)}$$

The above equations represent an approximate, closed-form, analytical solution to the subject problem. The amount of effort required to compute results with these solutions is at least an order of magnitude less than the effort required for rigorous numerical eration of the exact integro-differential equation.

RESULTS

A number of sample situations were computed, both for the special case and for the general case, and some results are presented in Figures 2 to 8.

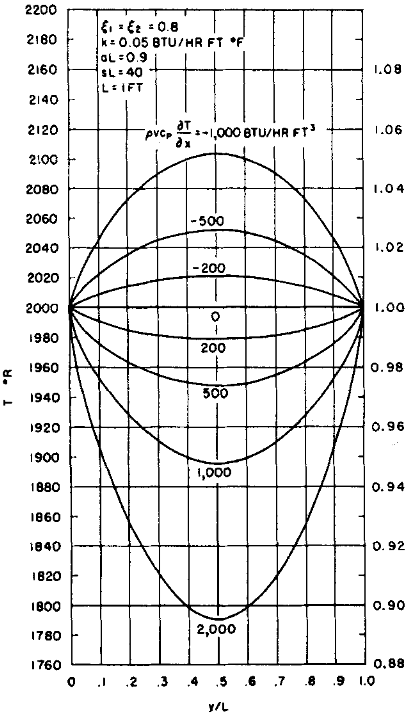


Fig. 3. Effect of convection on temperature profile.

For the special case, with one plate at temperature T_1 and a second plate at temperature T_2, separated by an absorbing-emitting medium of thickness L and absorption cross section a, the radiative heat transfer is uniquely determined by the two parameters $\sigma(T_1^4 - T_2^4)$ and (aL). Equation (32) gives an approximate, closed-form representation of this relationship. Figure 2 shows a plot of Equation (32) compared with the exact numerical solution obtained by Usiskin and Sparrow (*10*) as reported by Meghreblian (*11*). Also shown on the figure is a curve representing an approximate solution obtained by Meghreblian (*11*), with a uniform temperature in the medium assumed. It is seen that the simple relationship expressed by Equation (32) does a very credible job of representing the true situation. It is an improvement over the uniform-temperature approximation, especially at high optical thickness (aL). The heat transfer rates calculated by Equation (32) are greater than the true rates, a consequence of using two discrete fluxes to represent what is actually a diffuse flux field. The error is seen to be minimum for either optically thin or optically thick media.

Figures 3, 4, and 5 show some results for the general case when the two walls are at the same temperature and net heat transfer occurs between the flowing medium and the walls. The values of k, a, and s used in these sample calculations are approximations of the physical properties for a medium such as air carrying a suspension of dust or mist particles. Figure 3 shows the effect of the convective term $\rho v C_p \dfrac{\partial T}{\partial x}$ on the temperature profile. For negative values of the convective term, heat is being transferred from the medium to the walls and the temperature profiles are convex upward. For positive values of the term, the profiles are concave downward, representing heat transfer from the walls to the medium. Figure 4 shows the variation of the conductive, radiative, and total heat fluxes across the medium for one case where there is relatively high rate of heat transfer to the medium. The most notable feature is that a major fraction of the total heat transfer occurs by radiation. It is also seen that for this sample case the conductive transfer decreases sharply near the walls, while the radiative transfer decreases at almost a uniform rate to the center of the duct. Figure 5 further illustrates the importance of radiation to total heat trans-

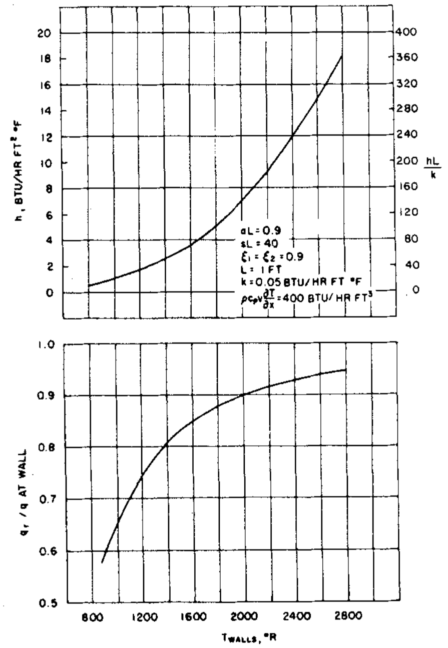


Fig. 5. Effect of temperature on heat transfer.

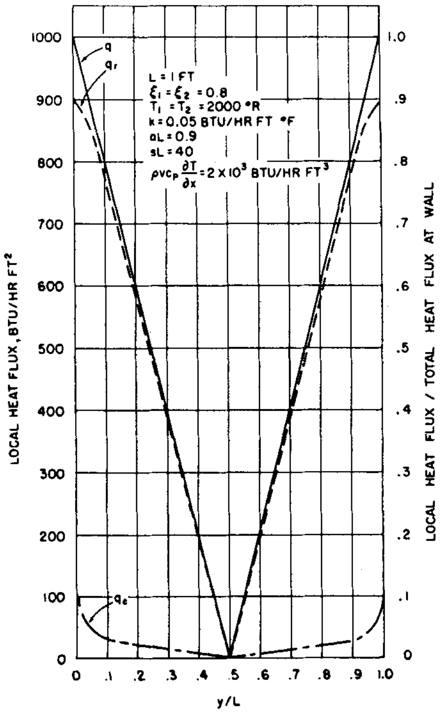


Fig. 4. Profiles of local heat fluxes.

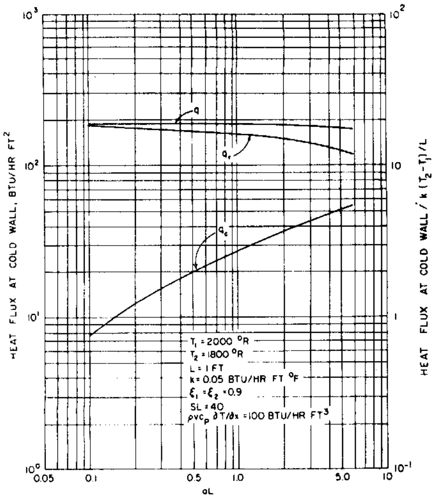


Fig. 6. Effect of absorption coefficient on heat transfer to cold wall.

being converted into sensible heat and raising the temperature of the elemental volume. This leads to a steeper temperature gradient in the medium near the cold wall and a higher rate of conductive heat transfer. As shown in Figure 6 the net effect is that the total rate of heat transfer is almost constant, decreasing by only 8% as (aL) increases by a factor of 60.

The effect of increasing the scattering coefficient is quite different, as can be seen from Figure 7. Radiant energy which is scattered backwards does not contribute any sensible heat to the elemental volume doing the scattering. Furthermore as the fraction of radiant energy leaving the hot wall which is scattered back increases, the amount of energy reaching the colder region and thus the amount of energy available for absorption is decreased. This leads to a less steep temperature gradient in the vicinity of the cold wall. Thus an increase in sL leads to a decrease in both q_r and q_c, so that total heat transfer decreases sharply. For the case illustrated in Figure 7 the total rate of heat transfer to the cold wall decreases by 77% as sL increases by a factor of 20. These two graphs give a good illustration of the fact that in simultaneous

fer. Here the temperature of the walls are varied, while net heat transfer to the medium is held constant. Since conductive heat transfer is independent of temperature while radiative heat transfer increases approximately as the cube of the absolute temperature, it is expected that the ratio of q_r/q would increase with temperature. For the case illustrated in Figure 5 it is seen that the radiant contribution to total heat transfer at the walls increases from 60% at a temperature of 900°R. to 95% at a temperature of 2,800°R. Figure 5 also shows that the effective heat transfer coefficient, defined as $h = q/(T_{\text{wall}} - T_{\text{bulk}})$, increases with increasing temperature, as would be expected.

Figures 6 through 9 show results for cases where the two walls are at different temperatures so that net heat transfer can occur from wall to wall as well as from wall to fluid. Figures 6 and 7 show the effects of the attenuation coefficients aL and sL on net heat transfer to the cold wall. As the absorption coefficient increases, radiant transfer decreases since the fraction of radiant energy absorbed by each elemental volume increases. Part of this absorbed energy is re-emitted as thermal radiation, the remainder

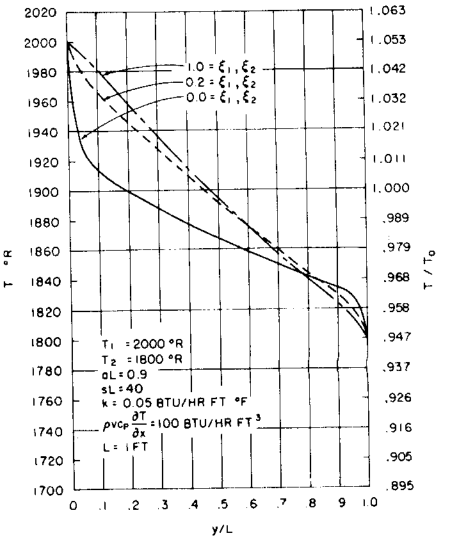


Fig. 8. Effect of wall emissivities on temperature profile.

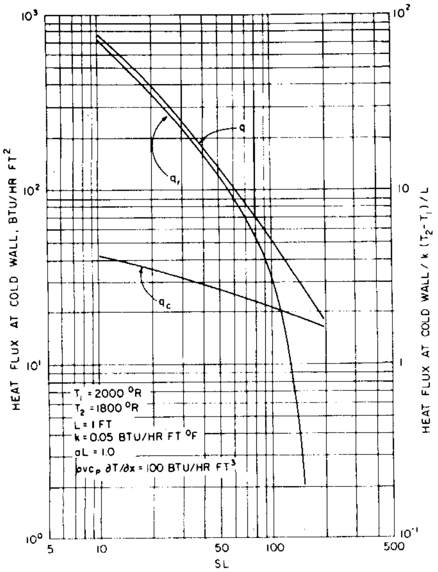


Fig. 7. Effect of scattering coefficient on heat transfer to cold wall.

radiative and convective heat transfer the attenuation due to scattering often can be much more important than the attenuation due to absorption.

Figure 8 illustrates the effect which wall emissivities have on the temperature profile within the medium. For a given case the continuity equation requires that the heat flux at any point y be sufficient to supply the amount of heat carried away in the region $(L - y)$ plus the amount of heat transferred to the cold wall. Therefore in a region where radiative transfer is low, the corresponding conductive transfer must be high; that is, the temperature gradient must be steep. This is the situation near the boundaries when wall emissivities are low. As shown in Figure 8 the result is that the temperature profiles become more distinctly S shaped as wall emissivities approach zero.

Figure 9 presents a comparison of the approximate solution with the numerical solution of Einstein (5) for a limiting case with no flow and no scattering. The temperature profile across the medium is plotted with the solid line representing Einstein's solution and the points representing the approximate solution. This sample calculation

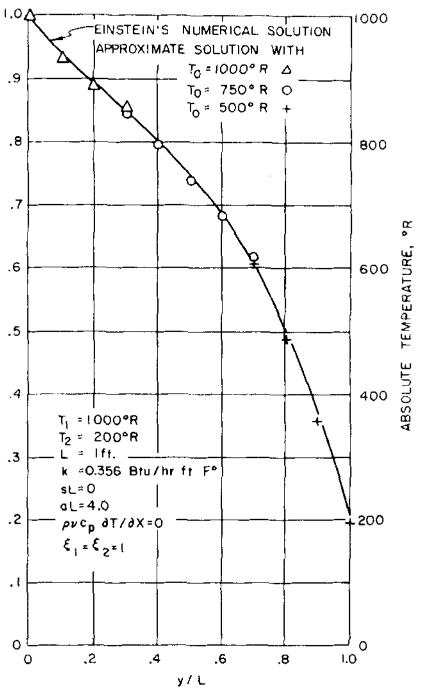


Fig. 9. Comparison of approximate and numerical solutions for case of no flow or scattering.

also serves to illustrate a method of improving the accuracy of the approximate solution by the use of different values for the arbitrary temperature T_o. By changing the value of T_o such that $0.4 < (T/T_o) < 1.3$ at any point in the medium, the truncation error inherent in Equation (33) is diminished. It is seen that the resulting approximate solution is in very good agreement with the numerical solution.

SUMMARY

The problem of simultaneous radiative and convective heat transfer to a medium in slug flow between infinite, parallel plates was formulated in terms of the two-flux model of Hamaker. Absorption, emission, and scattering within the medium as well as different wall temperatures and emissivities were taken into account. The nonlinear temperature term was represented by a truncated Taylor series, thus allowing an approximate, analytical solution to be obtained for the system of fourth-order differential equations.

Results were calculated for the limiting case of zero flow, conduction, and scattering. The calculated heat transfer rates were slightly higher than the exact values obtained by Usiskin and Sparrow from numerical iteration.

Sample computations were also made for the general case where flow, conduction, absorption, emission; and scattering occur. The effects on temperature profile and on heat fluxes were examined. It was found that for a medium with properties approximately representative of those for air carrying a suspension of dust or mist, radiation can account for 60 to 95% of total heat transfer in the temperature range of 900° to 2,800°R. It was also found that for some situations the effect of radiant scattering can be much more important than the effect of radiant absorption and emission.

It should be noted that the use of discrete fluxes and of a truncated Taylor series representation for the temperature term does introduce some inexactness into the results. However for many applications where only engineering precision is required, the convenience of a closed-form solution may make this approach more desirable than lengthy numerical iterations.

ACKNOWLEDGMENT

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NOTATION

a = absorption cross section, sq. ft./cu. ft.
A_1, A_2, A_3, A_4 = boundary constants
C = $\rho C_p v \dfrac{\partial T}{\partial x}$ (B.t.u.)/(hr.)(cu. ft.)
G = net heat generation per unit volume, (B.t.u.)/(hr.)(cu. ft.)
h = heat transfer coefficient, (B.t.u.)/(hr.)(sq. ft.)(°F.)
I = forward radiant flux in y direction, (B.t.u.)/(hr.)(sq. ft.)
i_+, i_-, i_n = fluxes of radiant energy as defined by Equations (1), (2), and (3) respectively, (B.t.u.)/(hr.)(sq. ft.)
i_λ = monochromatic intensity of radiant energy flux, (B.t.u.)/(hr.)(sq. ft.)(unit λ)(unit steradian)
J = backward radiant flux in y direction, (B.t.u.)/(hr.)(sq. ft.)
k = thermal conductivity, (B.t.u.)/(hr.)(ft.)(°F.)
L = distance between walls, ft.
$\vec{n}$ = normal vector
q = total heat flux, (B.t.u.)/(hr.)(sq. ft.)
q_c = conductive heat flux, (B.t.u.)/(hr.)(sq. ft.)
q_r = radiative heat flux, (B.t.u.)/(hr.)(sq. ft.)
s = backscattering cross section, sq. ft./cu. ft.
T = absolute temperature, °R.
T_o = arbitrary temperature, used in Taylor expansion
v = velocity, ft./hr.

Greek Letters

α = defined by Equation (42)
β = defined by Equation (43)
γ = defined by Equation (37)
δ = defined by Equation (38)
ϵ_1, ϵ_2 = emissivities of walls at $y = 0$ and $y = L$, respectively
σ = Stefan-Boltzmann constant, 1.713×10^{-9}, (B.t.u.)/(hr.)(sq. ft.)(°R.)(2π steradians)

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